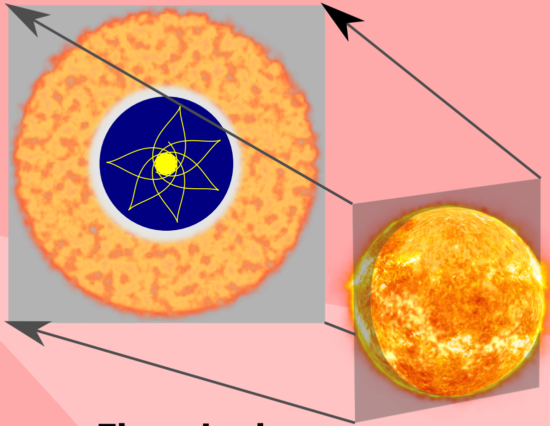


PUTTING A DAMPER ON THINGS:

Mode depression in red giant stars

and how **ray tracing** might **shed some light** on the role of *magnetic fields...*

UNIVERSITY OF CAMBRIDGE

This poster was designed and presents research done by Dr. Cleo (Shyeh Tjing) Loi , a Junior Research Fellow in Astrophysics at Churchill College, Cambridge University. She currently works in the Astrophysical Fluid Dynamics Group within the Department of Applied Mathematics and Theoretical Physics (DAMTP), from which she received a PhD in 2019, for her thesis entitled *'Magnetic fields and stellar oscillations'*.

BACKGROUND

The study of stellar oscillations

Stars are self-gravitating fluid bodies in which waves can propagate. These waves may be excited through ***convection***, ***flares*** and ***global instabilities***. Constructive interference produces standing waves, i.e. global modes of oscillation. These can be detected by monitoring fluctuations in the brightness of a star, which arise from surface temperature fluctuations due to the fluid motions. Properties such as the ***frequencies***, ***amplitudes*** and ***widths*** of peaks in the power spectrum contain information about a star's material properties. The study of stellar interiors by analysing their free modes of oscillation is called ***asteroseismology***, and relies on a good theoretical understanding of the fluid behaviour of stars.

Mode depression: an ongoing mystery

Amongst the 10,000+ red giant stars observed by the *Kepler* space telescope, it was found that in ~20% of these, their non-radial modes ***have amplitudes significantly lower*** than the rest of the population[1]. Radial modes appear not to be affected. This phenomenon is restricted to those stars massive enough to have ***previously had convective cores*** when on the main sequence[2]. With convection closely linked to ***dynamo action***, and non-radial modes being those ***localised to the core*** (rather than the envelope, as for radial modes), this strongly hints at the possible role of a ***hidden, leftover magnetic field***. However, preliminary magnetic theories predict 100% suppression of the core portion of affected modes[3], which is inconsistent with observations[4]. Rather, they show that ***depression is only partial***.

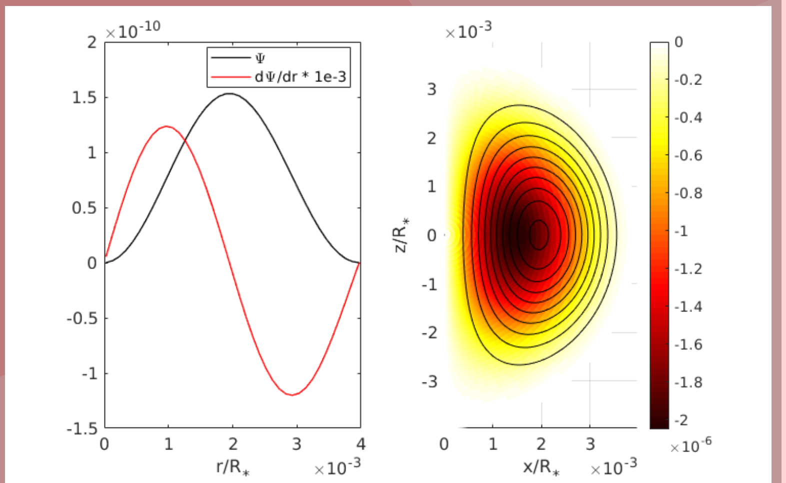


Figure 1. Magnetic field configuration

Twisted-torus magnetic fields

This work aims to study the problem of wave damping for realistic stellar models. The field configuration used here is an axisymmetric twisted torus, believed to be a good description of a ***magnetic equilibrium***[5]. This is shown in Fig. 1, where the flux function

$$\psi(r,\theta) = \Psi(r)\sin^2\theta$$

defines the field according to

$$\mathbf{B} = \frac{1}{r\sin\theta}\nabla\psi\times\hat{\boldsymbol{\phi}} - \frac{\lambda\psi}{r\sin\theta}\hat{\boldsymbol{\phi}}\ .$$

METHODS

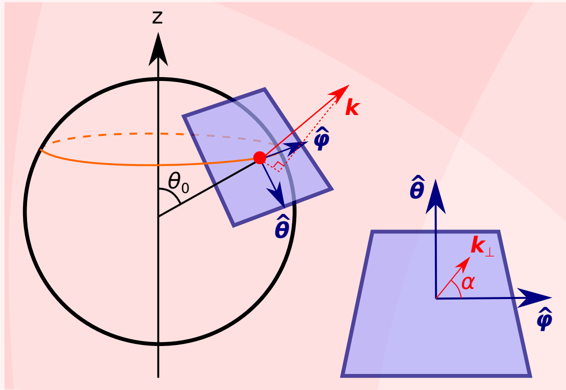


Figure 2. Ray launching parameters

Hamiltonian ray tracing

Wave propagation was modelled using Hamiltonian ray tracing, which calculates the ***path of wave packets*** launched in a system with known background properties. The time evolution of the position, **x**, and conjugate momentum (i.e. wavevector), ***k***, are given by solving Hamilton's equations

$$\frac{d\mathbf{x}}{dt} = \nabla_{\mathbf{k}}H\ , \qquad \frac{d\mathbf{k}}{dt} = -\nabla H\ ,$$

where the Hamiltonian, $H = H(\mathbf{x},\mathbf{k})$, is given by the wave dispersion relation, $\omega = \omega(\mathbf{x},\mathbf{k})$. For ***magneto-gravity waves***, this is

$$\omega^2 = (\mathbf{k}\cdot\mathbf{v}_A)^2 + \kappa_\perp^2 N^2 .$$

For each stellar model, 1200 gravity rays were launched into the core, from 30 different colatitudes, θ_0, and with 40 different initial polarisations, α (see Fig. 2).

RESULTS

Magnetic field?

NO

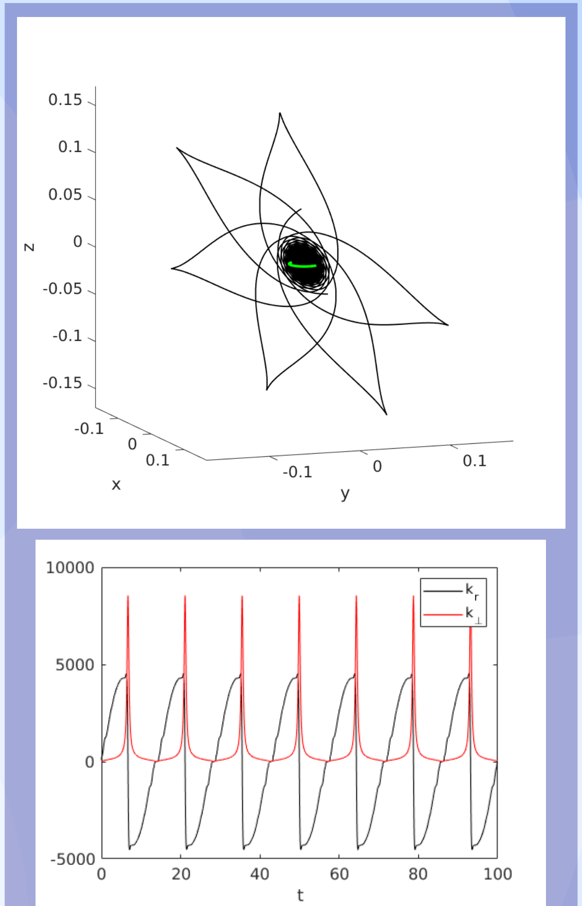


Figure 3. Ray trajectory (B = 0)

Ray path lies in a single plane (top), as expected for a spherically symmetric Hamiltonian. The wave packet undergoes periodic bouncing between inner and outer turning points. Wavevector components (bottom) remain bounded and periodic.

YES

two possible outcomes...

Trapping

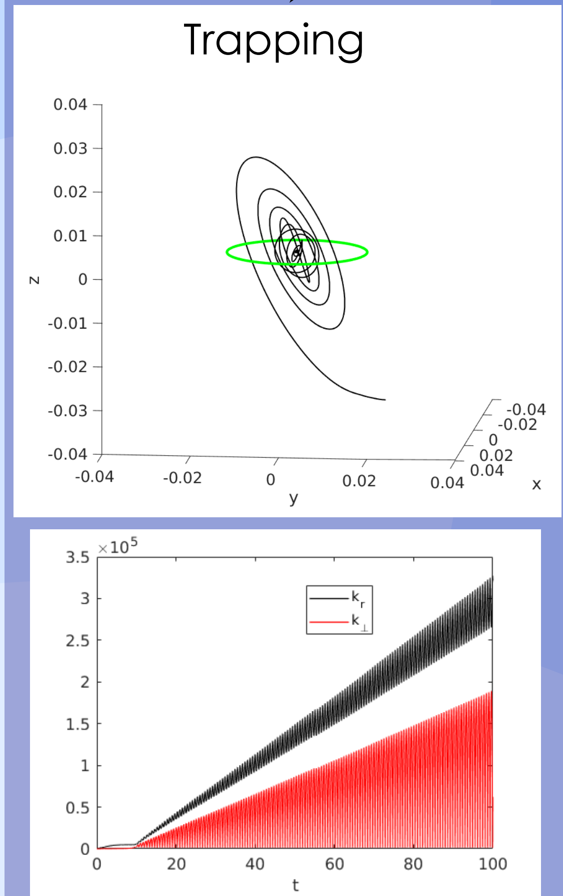


Figure 4. Ray trajectory (B > 0)

$\alpha = 58.5°$
$\theta_0 = 141°$

Ray path does not lie in a single plane, with out-of-plane motion occurring where the magnetic field is non-zero. The extent of the magnetised region is indicated with a green circle. Here the ray spirals irreversibly inwards, its group velocity decreasing and wavenumber diverging continuously.

All other launch parameters identical

Reflection

$\alpha = 40.5°$
$\theta_0 = 99°$

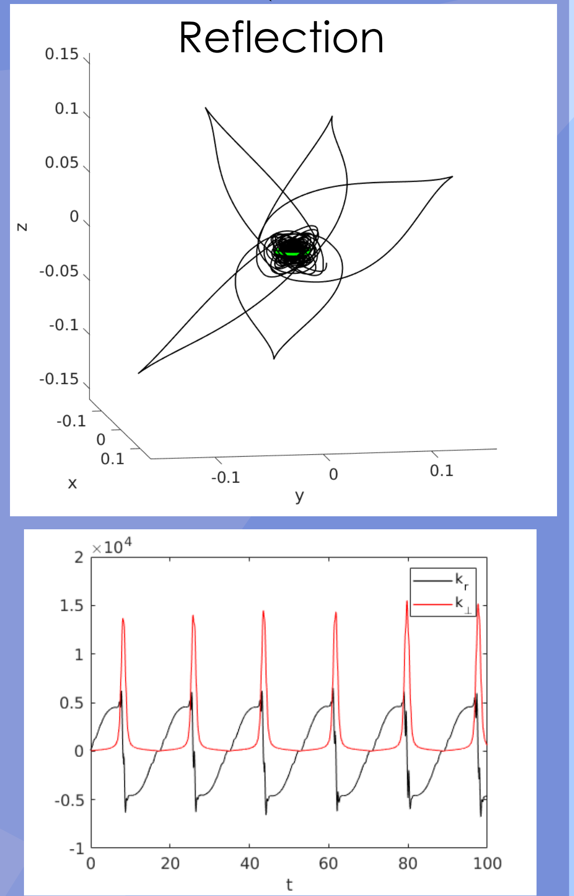


Figure 5. Ray trajectory (B > 0)

Ray path also does not lie in a single plane, but otherwise is qualitatively similar to the case where B = 0 (Fig. 3). Motion takes the form of quasi-periodic reflections, with bounded wavevector. Importantly, the only differences compared to Fig. 4 (trapped ray) are the launch values of θ_0 and α. Field strength/configuration, wave frequency, background stellar model etc. are identical.

DISCUSSION

When will magnetic fields be important?

With rotation neglected, the three main types of wave in a magnetised star are ***acoustic***, ***Alfvén*** and ***gravity*** waves. Deep in red giant cores, fluctuations mainly take the form of gravity waves, while in the envelope, fluctuations tend to be acoustic in nature. In general, two wave modes will interact when their frequencies ω and wavenumbers k coincide, i.e. a ***resonance condition*** is met. The dispersion relations $\omega = \omega(k)$ for the three wave modes are shown in the box below. While acoustic and Alfvén waves will only interact when the plasma beta $c_s/v_A \sim O(1)$, thus requiring a minimum field strength, ***gravity and Alfvén waves will always interact in some part of parameter space***, regardless of the field strength. This is given by the intersection point between the curves in the right panel of Fig. 6, and occurs at lower frequencies when the field is weaker. Solving for this intersection point gives the result that gravity waves with a given ω will resonantly interact with a magnetic field when the Alfvén speed reaches

Acoustic	$\omega = kc_s$
Alfvén	$\omega = \mathbf{k}\cdot\mathbf{v}_A$
Gravity	$\omega = \kappa_\perp N$

$$v_{A,\mathrm{crit}} \sim \frac{r}{N}\frac{\omega^2}{\sqrt{l(l+1)}} .$$

This defines the concept of a ***critical surface***, which, for a given field configuration and set of wave parameters, corresponds to the locus of points satisfying the resonance criterion. There, ***mode conversion*** may occur.

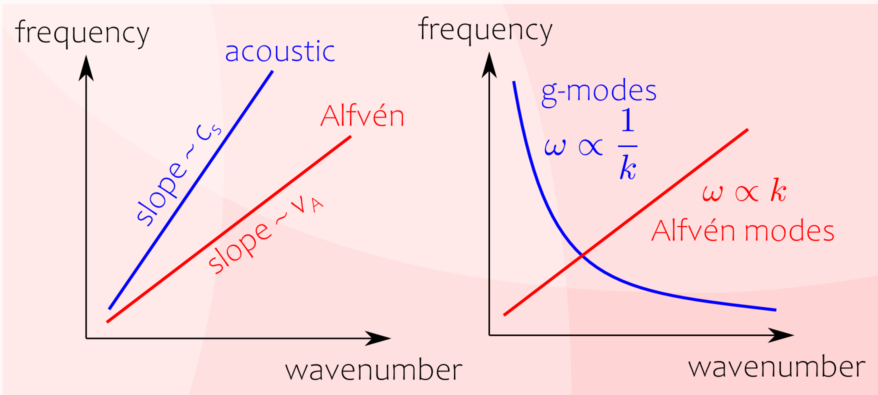


Figure 6. Comparison of wave dispersion relations

Why are some rays trapped while others reflected?

The key feature distinguishing trapped rays from reflected ones lies in their time-dependent behaviour of the gravity and Alfvén contributions to the total wave frequency. These are shown in red and blue, respectively, in Fig. 7. For ***trapped rays***, the red and blue envelopes strongly overlap, indicating ***acquisition of substantial Alfvén character***. Thus trapped rays are those that ***encounter a critical surface***, i.e. the resonance criterion is satisfied somewhere along their trajectory.

However, for ***reflected rays*** the two envelopes do not overlap, and the ***gravity component always dominates***. Thus reflected rays are those that are ***able to avoid critical surfaces***. How this might be possible can be deduced from Fig. 8, which shows the 1200 launch points, colour-coded by outcome: blue for trapped rays and red for reflected. Different θ_0 are plotted on different latitudes, while different α are plotted on different longitudes. Hence reflected rays are those that are ***equatorially orbiting*** with ***zonal polarisation vectors***. These are able to minimise the Alfvén component by having near-perpendicular $\boldsymbol{k}$ and $\boldsymbol{v}_A$.

RESULTS

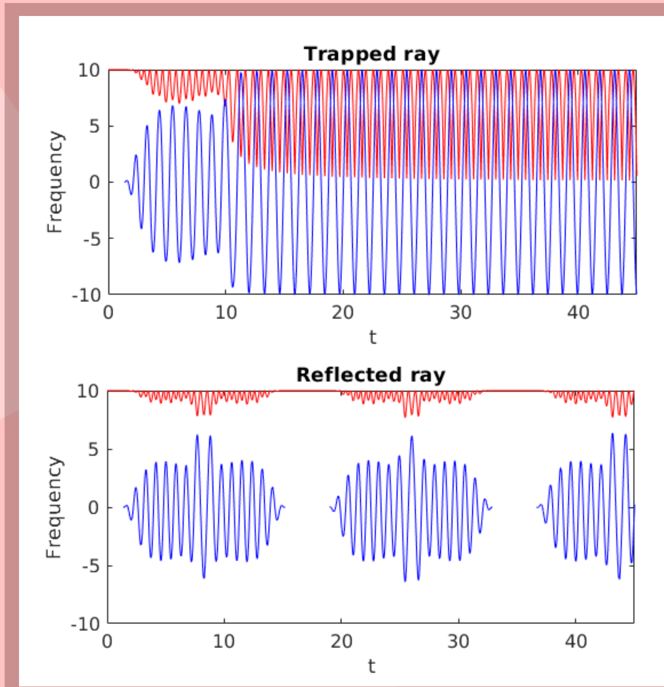


Figure 7. Gravity and Alfvén frequencies

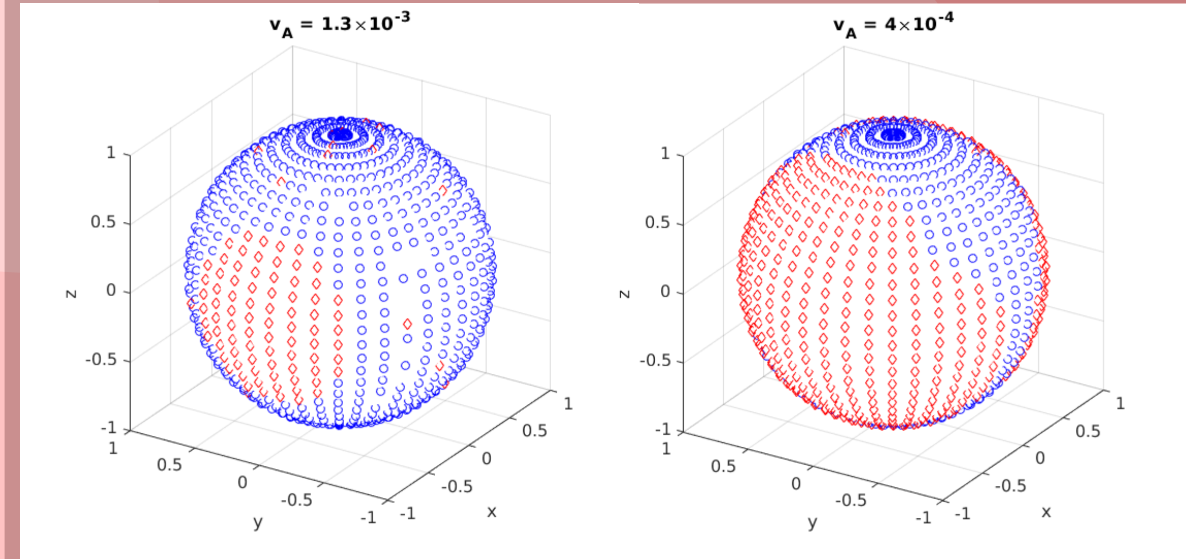


Figure 8. Distribution of trapped (blue) and reflected (red) rays

DISCUSSION

Comparison with observations

The loss of energy via trapping causes ***damping*** of the modes, at a rate proportional to the area of the sphere covered by blue points in Fig. 8, which we call the ***trapping fraction***, f_T. This is a function of many parameters, including the field strength, wave frequency ω and the spherical harmonic degree l (Fig. 9, left). Measurements of f_T using the ray tracing method suggest that trapping/damping rates are higher at lower frequencies, ***predicting the existence of stars with a positive gradient in mode amplitudes*** with frequency. Several of these are known in the literature, e.g. KIC 6975038 (Fig. 9, right). For more quantitative comparisons, note that the observable quantity is the visibility (V^2) ratio between depressed and normal modes, given in terms of the convective and magnetic damping times τ_c ~ 15 d and τ_m by

$$\frac{V^2_{\rm depressed}}{V^2_{\rm normal}} = \frac{1}{1+\tau_c/\tau_m}, \text{ where } \tau_m = \frac{1}{\nu_{\rm dyn} T^2 f_T},$$

$$T = \exp\left(\int_{r_g}^{r_p} \mathrm{i}k_r \mathrm{d}r\right) \text{ and } \nu_{\rm dyn} = \frac{1}{2\pi}\sqrt{\frac{GM_*}{R_*^3}}.$$

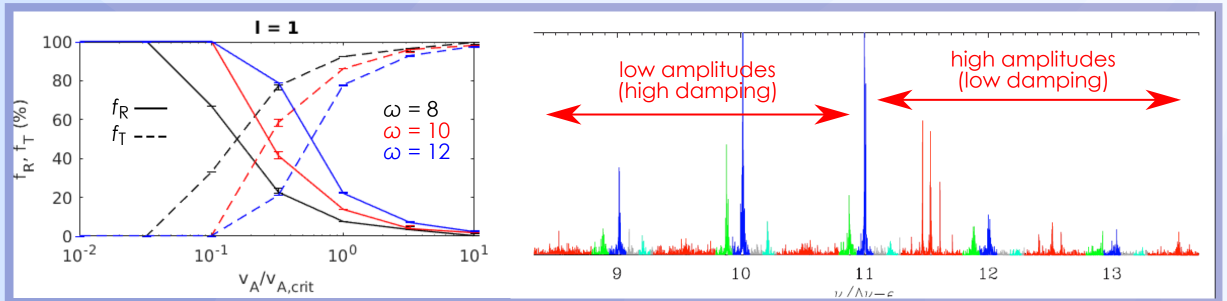


Figure 9. Variation with frequency (right panel from ref. 4)

Table 1. Theoretical predictions

SH degree	l = 1	l = 2	l = 3
T^2	0.4	0.03	10^{-3}
f_T	0.55	0.65	0.7
τ_m (d)	5.6	64	1.8×10^3
V^2_{dep}/V^2_{norm}	0.27	0.81	0.99

Measured/calculated values of relevant quantities are shown in Table 1, along with the predicted V^2 ratio, for a $2M_\odot$ red giant model generated using the stellar evolutionary code '*Modules for Experiments in Stellar Astrophysics*[6]'. Values are shown for different spherical harmonic degrees, and the results appear to be ***quantitatively consistent with observations***, as can be seen in Fig. 10 which shows the V^2 values of the two populations of stars, for l=1. Horizontal lines have been drawn at the characteristic value of the normal population (white), and predicted value of the depressed population (yellow) according to Table 1. The yellow pentagram indicates the ν_{max} value of the stellar model considered.

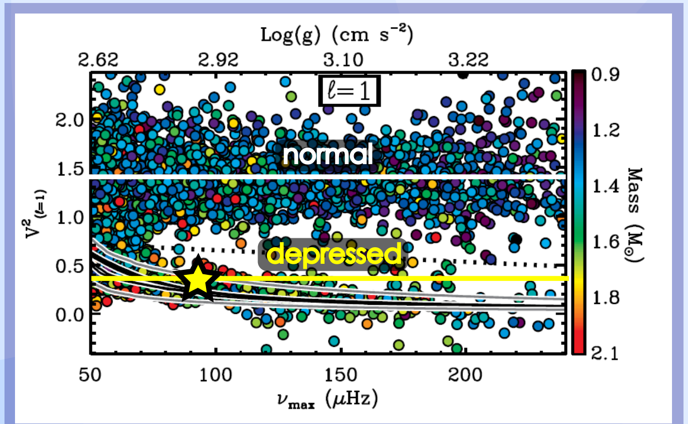
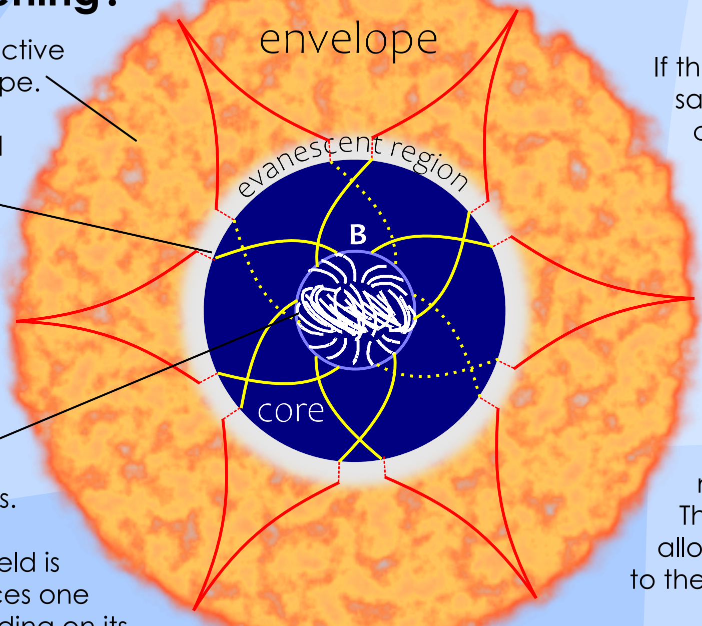


Figure 10. Observed dichotomy (ref. 2)

What is happening?



Waves are excited by convective motions in the envelope.

A fraction of these tunnel through the evanescent region and resume propagation in the stably stratified core.

If there is no magnetic field, the wave reflects near the centre and returns to the envelope, reinforcing the standing wave pattern. This gives rise to "normal" stars.

If a strong magnetic field is present, the wave experiences one of two outcomes, depending on its polarisation and where it impacts the magnetised region.

If the resonance criterion is satisfied, i.e. the wave meets a critical surface, it acquires significant Alfvén character and becomes trapped. Its wavenumber diverges and the energy is dissipated at small scales by viscosity/resistivity.

However, this work has shown that even with a strong field, there exist trajectories that never meet a critical surface. These waves are reflected, allowing for partial energy return to the surface.

This partial energy return is required to explain observations of partial depression.

SUMMARY

For more details, please see the journal article: '**Magneto-gravity wave packet dynamics in strongly magnetized cores of evolved stars**', by Shyeh Tjing Loi (2020), *Monthly Notices of the Royal Astronomical Society*, vol. 493, no. 4, pp. 5726-5742

References
1. Mosser et al. (2012), *A&A*, 537, A30
2. Stello et al. (2016), *Nature*, 529, 364
3. Fuller et al. (2015), *Science*, 350, 423
4. Mosser et al. (2017), *A&A*, 598, A62
5. Braithwaite & Nordlund (2006), *A&A*, 450, 1077
6. Paxton et al. (2011), *ApJSS*, 192, 3